Optimization of mechanical draft counter flow wet-cooling towers using a rigorous model

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ABSTRACT

In this paper, an optimal design algorithm for mechanical draft counter flow wet-cooling towers based on the rigorous Poppe model and mixed-integer nonlinear programming (MINLP) is presented. Unlike the widely used Merkel method, the Poppe model takes into consideration the effects of the water loss by evaporation and the nonunity of the Lewis factor. As a result, the Poppe model is able to predict the performance of wet-cooling towers very accurately compared to the Merkel method. The optimization problem is formulated as an MINLP model by considering all the mass and energy balances, equations for physical properties, and empirical correlations for the loss and overall mass transfer coefficients in the packing region of the tower, in addition to feasibility constraints. The objective function to be minimized is the total annual cost, which includes capital and operating costs. The mathematical programming problem is solved with the GAMS software. Six case studies are used to show the application of the proposed algorithm. The case studies demonstrate that there can be large differences between the optimal designs based on the Poppe method and the Merkel method.

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1. Introduction

Along to the industrial history, the process engineers have looked for strategies and methodologies to minimize the process costs and to increase the profits. In this area, the mass [1] and thermal water integration [2,3] have represented an important role. Regarding to the thermal water integration, several strategies have been reported around the open re-circulating cooling water systems, because they are widely used to dissipate the low-grade heat of chemical and petrochemical process industries, electric-power generating stations, refrigeration and air conditioning plants. In all these systems, water is used to cool down the hot process streams, and then this water is cooled by evaporation and direct contact with air in a wet-cooling tower and recycled to the cooling network. Therefore, the cooling towers are very important industrial components and there are many references that present the fundamentals to understand these units [4–6].

The first practical theory of counter flow cooling towers was developed by Merkel [7]. In this theory, the water loss due to evaporation and the water-film heat-transfer resistance are neglected. In addition, a Lewis factor for moist air of unity is assumed. These assumptions allow the differential equations of the rigorous model for simultaneous heat and mass transfer processes occurring in cooling towers to be reduced to a single separable differential equation in terms of air enthalpy difference as the driving force. This method gives only the exit temperature of the water stream and the exit enthalpy of the air stream when it is provided with the inlet air and water conditions. In order to obtain the temperature and humidity of the outlet air, Merkel assumed that the air leaving the cooling tower is saturated with water vapor [6].

It is possible to extend Merkel’s enthalpy theory to include a finite liquid-side film resistance to heat transfer [8–12]. For typical operating conditions, however, the local bulk temperature of the water is seldom more than 0.3 K above the temperature of the air at the air–water interface [4]. Therefore, for the purpose of cooling-tower analysis it is safe to neglect the water-film resistance [4,5]. Over the years there have been a number of contributions on the literature for cooling towers that are based upon this assumption. Jaber and Webb [13] developed an effectiveness-number of transfer units (e-NTU) approach by utilizing the assumption of a linearized air saturation enthalpy. The effect of nonlinearity of the
equilibrium enthalpy was considered by using an enthalpy correction factor in the effectiveness definition only for the cases with smaller air flow heat capacity rate. However, like Merkel, they neglected water evaporation loss and assumed that the Lewis factor was equal to unity. Nahavandi et al. [14], Osterle [15] and Braun et al. [16] proposed models that include the effect of the mass of water loss by evaporation on energy balance. Consequently, unlike the Merkel method, these models provide the state of the air exiting the tower, not just its enthalpy, along with a more accurate value of the required Merkel number (which is similar to number of transfer units – NTU). However, their results are only applicable for Lewis factors equal to unity.

Sutherland [17] proposed a model that includes the effect of water loss by evaporation and sets the Lewis factor equal to 0.9. Comparing his results with those obtained from the Merkel method, he found that mechanical draught cooling towers can be undersized between 5% and 15% through the use of the Merkel method. Kloppers and Kröger [18] stated that Lewis factor may deviate significantly from unity and can be in the range from 0.5 to 1.3 for wet-cooling towers. Poppe and Rögener [19] developed a more complete and accurate model of a cooling tower, which is commonly known as the Poppe method [18]. This method considers the effects of Lewis factor and water evaporation on the air process states along the vertical length of the tower. The Lewis factor for air/water mixtures is obtained from the equation given by Bosnjakovic [20]. The Poppe method also accounts for the possibility of supersaturation of the moist air during the heat and mass transfer processes. In this method, the Merkel number for the cooling tower is obtained by solving numerically three simultaneous differential equations governing heat and mass transfer and air flow in the tower fill. Kloppers and Kröger [18] have used the Poppe method to investigate the effect of the Lewis factor on performance prediction of natural draft and mechanical draft wet-cooling towers. They found that as the Lewis factor increases, the heat rejection rate from the tower increases, with a corresponding increase in air outlet temperature and a decrease in the water outlet temperature. Also, they found that the influence of the Lewis factor diminishes when the inlet ambient temperature is relatively high.

More recently, Ren [21] presented an analytical solution of a detailed model for counter flow wet-cooling towers, which takes into consideration the effects of Lewis factor and water evaporation as well as the heat-transfer resistance in the air–water interface. However, he used a linear function of the surface temperature for calculating the humidity ratio of air in equilibrium with the water surface. In a later work, Ren [22] used the perturbation technique to develop another analytical solution for evaluating the thermal performance of wet-cooling towers, but taking into consideration the effect of non-linearities of humidity ratio and enthalpy of air in equilibrium with water.

The Merkel [7], e-NTU [13] and the rigorous Poppe [18] methods are the most popular approaches to address design and rating calculations of cooling towers. Kloppers and Kröger [6,23] have discussed the differences between cooling tower results obtained with the rigorous Poppe method and Merkel and e-NTU approximate methods, which are more frequently used because of its simplicity. This comparative study shows that all methods predict practically identical water outlet temperatures for mechanical and natural draft towers; however, the computation of the water outlet temperature is improved if the water loss by evaporation is considered. Kloppers and Kröger [6] also made a comparison between a large number of measured air outlet temperatures in the fill test facility at the University of Stellenbosch and the predictions from Merkel, e-NTU, and Poppe approaches. The comparison results show that the measured outlet air temperatures and the values obtained by the Poppe method are in excellent agreement. However, the Merkel and e-NTU methods do not predict the air outlet temperature very accurately compared to the Poppe method. On the other hand, Bourillot [24,25] found that the values of evaporated water flow rate calculated from the Poppe method agree well with available full scale cooling-tower test results. The Poppe method, therefore, is an experimentally proven detailed model for reliable representation of wet-cooling towers that should be employed for design applications where more accurate results are needed.

The optimal design of cooling towers is an industrially relevant problem, in which a number of continuous and discrete variables must be considered simultaneously such as the wet-bulb temperature of the ambient air, water-to-air mass ratio, water outlet temperature, water inlet temperature, water loss by evaporation, fresh water consumption, type of draft, type of packing, tower fill height, and cross-sectional area of the tower [26]. Also, the optimal design of cooling towers is subjected to a number of feasibility constraints such as minimum and maximum values of water-to-air mass ratios, water and air mass velocities, and so on. Past studies [26–29] to obtain the optimal economical design of counter flow wet-cooling towers are based on the Merkel or e-NTU approximate methods. Consequently, they may lead to sub-optimal designs and possible unreliable model predictions due to the errors introduced by neglecting the water loss by evaporation and assuming a Lewis factor of unity.

The objective of this paper is to use the Poppe method [18] for the economical optimization of mechanical draft counter flow wet-cooling towers. The optimization problem is formulated as a MINLP problem, which can be represented and optimized with GAMS/DICOPT [30]. The optimization is to be performed over the design and operating variables including water-to-air mass ratio, air mass flow rate, water inlet and outlet temperatures, tower temperature approach, type of packing, height and area of the tower packing, total pressure drop of air flow, power consumption of the fan, fresh water consumption, outlet air conditions and Merkel number. The mass transfer and pressure drop characteristics of the types of packing considered are modeled with the empirical correlations obtained from experimental measurements by Kloppers and Kröger [31,32]. The objective function is defined as the minimization of the total annual cost that includes the water consumption cost, power fan cost and capital cost of the cooling tower.

Six examples of application of the developed MINLP model are reported. Comparison of the so obtained results with those obtained by an approximate model based on the Merkel method, which has been fully presented by Serna-González et al. [26], shows that large differences may arise between the optimal designs of the Poppe and Merkel methods. These results, as well as a description of the model equations and numerical methods that form the basis for the proposed MINLP formulation, are given below.

2. Problem statement

The problem addressed in this paper can be stated as follows. Given are the heat load to be removed in the cooling tower, the dry- and wet-bulb temperatures of the inlet air, the lower and upper limits for the water outlet temperature and for the water inlet temperature, the minimum temperature approach, the minimum difference between the dry-bulb and wet-bulb temperatures at each integration interval to avoid the saturation of passing air through the fill, the fan efficiency as well as the economical information such as unit cost of electricity, unit cost of fresh water, fixed cooling-tower cost, incremental cooling-tower cost based on the air mass flow rate and yearly operating time. The problem then consists in determining the design details (make-up water consumption, air mass flow rate, state of the outlet air, type of
packing, packing height and tower cross-sectional area, cooling-tower approach and range, total pressure drop of the air stream, power consumption of the tower fan, and Merkel number) of the counter flow cooling tower that satisfy the cooling requirements with a minimum total annual cost.

3. Model formulation

The major equations for the heat and mass transfer in the fill section and the design equations for the cooling tower are described in this section. The subscripts used in the model formulation are defined first: in (inlet), out (outlet), j (constants to calculate the transfer coefficient), k (constants to calculate the loss coefficient), r (make-up), ev (evaporated water), d (drift), b (blowdown), m (average), w (water), a (dry-air), wb (wet-bulb), n (interval of fill), fi (fill), fr (cross-sectional), misc (miscellaneous), t (total), vp (velocity pressure), f (fan), ma (air–water mixture), e (electricity), s (saturated) and v (water vapor). In addition, the superscript i is used to denote the type of fill and the scalar NTI represents the top interval of the packing height. It should be noted that the parameters and variables of the model are represented by italic letters. The nomenclature section presents the definition of the variables used in the model and the model formulation is described as follows.

3.1. Heat and mass transfer in the fill section for unsaturated air

According to the mathematical development of the Poppe method given by Kloppers and Kröger [23], which is based on the control volumes in the fill of a counter flow wet-cooling tower shown in Figs. 1 and 2, the detailed steady-state mass and energy balance equations for unsaturated air can be written as

\[
\frac{dw}{dT_w} = \frac{cp_w m_w}{ima} (w_{s,w} - w) - \frac{cp_w T_w (w_{s,w} - w)}{ima - (w_{s,w} - w) k_v - (w_{s,w} - w) cp_w T_w} \tag{1}
\]

\[
\frac{dima}{dT_w} = m_a cp_w \left( 1 + \frac{cp_w T_w (w_{s,w} - w)}{ima - (w_{s,w} - w) k_v - (w_{s,w} - w) cp_w T_w} \right) \tag{2}
\]

and the transfer characteristic of the packing is given by

\[
\frac{dMe}{dT_w} = \frac{cp_w}{ima - (w_{s,w} - w) k_v - (w_{s,w} - w) cp_w T_w} \tag{3}
\]

In above differential equations, \( T_w \) is the local bulk water temperature, \( m_w \) is the liquid water mass flow rate, \( w \) is the mass-fraction humidity of the air stream, \( ima,s,w \) is the enthalpy of saturated air at evaluated at the local bulk water temperature, \( ima \) is the enthalpy of the air–water vapor mixture per unit mass of dry-air, \( k_v \) is the enthalpy of the water vapor, \( w_{s,w} \) is the saturation value of the mass-fraction humidity of air evaluated to the local bulk water temperature, \( cp_w \) is the specific heat at constant pressure of the liquid water, \( Me \) is the transfer coefficient or Merkel number according to the Poppe method, \( Lef \) is the Lewis factor, and \( m_a \) is the mass flow rate of dry-air through the tower.

The Lewis factor is a dimensionless group that is defined as \( Lef = h/l_{ev} cp_a \) and it is an indication of the relative rates of heat and mass transfer in an evaporative process. According to Bosnjakovic [20], the Lewis factor for unsaturated air can be expressed as:

\[
Lef = 0.865 \frac{1}{0.665} \left( \frac{w_{s,w} + 0.622}{w + 0.622} - 1 \right) \left( \ln \frac{w_{s,w} + 0.622}{w + 0.622} \right) \tag{4}
\]

Other properties for air, water and air–water mixtures used are obtained from correlations given in Appendix A.

The ratio of the mass flow rates \( m_w/m_a \) in Equation (1) and Equation (2) changes as the air moves toward the top of the fill. A mass balance for the control volume of a portion of the fill illustrated in Fig. 3 yields the ratio of the mass flow rates at any position in the tower:

\[
\frac{m_w}{m_a} = \frac{m_{w,in}}{m_a} \left( 1 - \frac{m_a}{m_{w,in}} (w_{out} - w) \right) \tag{5}
\]

where \( m_{w,in} \) is the water mass flow rate entering to the cooling tower and \( w_{out} \) is the mass-fraction humidity of the air leaving the cooling tower.
The Poppe model consists of the above set of coupled ordinary differential and algebraic equations, which can be solved simultaneously to provide the air humidity, the air enthalpy, the water temperature, the water mass flow rate and the Merkel number profiles in the cooling tower. Also, the state of the outlet air from the cooling tower can be fully determined by this model. The Merkel model can be derived from the Poppe model by assuming a Lewis factor of unity (Lef = 1) and negligible evaporation of water, i.e., \( dm_w = 0 \).

A model with ordinary differential equations and algebraic equations is quite complex for MINLP optimization purposes. Due to this fact, the set of ordinary differential equations comprising the Poppe model is converted into a set of nonlinear algebraic equations using a fourth-order Runge–Kutta algorithm. The final algebraic model is given in Appendix B.

Next disjunction and its reformulation through the convex hull technique \([33]\) is used for the optimal selection of fill type:

\[
\left[ \begin{array}{c} y^1 \\
\text{(splash fill)} \\
c^j = c^{j,1}, j = 1, \ldots, 5 \\
\end{array} \right] \lor \\
\left[ \begin{array}{c} y^2 \\
\text{(trickle fill)} \\
c^j = c^{j,2}, j = 1, \ldots, 5 \\
\end{array} \right] \lor \\
\left[ \begin{array}{c} y^3 \\
\text{(film fill)} \\
c^j = c^{j,3}, j = 1, \ldots, 5 \\
\end{array} \right]
\]

3.2. Design equations

In this section we present additional equations to complete the mathematical model for the optimization of counter flow wet-cooling towers such as the heat load definition, the overall mass balance for the water, empirical correlations for the loss and overall mass transfer coefficients in the packing region, and so on.

3.2.1. Heat load

The heat transferred from the water to the air stream (\( Q \)) is given by Equation (6), which does not neglect the water loss by evaporation

\[
Q = c_p w_{\text{in}} m_w w_{\text{in}} T_w w_{\text{in}} - c_p w_{\text{out}} m_w w_{\text{out}} T_w w_{\text{out}}
\]

where \( T_w w_{\text{in}} \) and \( T_w w_{\text{out}} \) are the inlet temperature and the outlet temperature of the water, respectively, \( c_p w_{\text{in}} \) and \( c_p w_{\text{out}} \) are the corresponding liquid specific heats at constant pressure, respectively, and \( m_w w_{\text{out}} \) is the water outlet mass flow rate. In the above equation \( c_p w \) is in \( \text{kJ/kg K} \) and \( T_w \) is in K.

An overall mass balance for the water gives

\[
m_w w_{\text{out}} = m_w w_{\text{in}} - m_a w (w_{\text{out}} - w_{\text{in}}) - m_{w d}
\]

(7)

where \( m_{w d} \) is the loss of water due to drift.

3.2.2. Transfer and loss coefficients

The overall transfer coefficient of a counter flow wet-cooling tower can be obtained using the following empirical correlation \([31]\) (whose correlation coefficient (\( r^2 \)) is given in Table 1 according to the fill type).

\[
M e_{\text{h-NTU}} = c_1 \left( \frac{m_w w_{\text{in}}}{A_f} \right) c_2 \left( \frac{m_a}{A_f} \right) c_3 \left( L_f \right)^{1+c_4} \left( T_w w_{\text{in}} \right)^{c_5}
\]

(8)

where \( A_f \) is the packing area, \( L_f \) is the height packing, \( c_1, c_2, c_3, c_4 \) and \( c_5 \) are constants that depend on the type of fill used, and \( m_w w_{\text{in}} \) is the average value of the water mass flow rate. Thus,

\[
m_w w_{\text{in}} = \frac{m_w w_{\text{in}} + m_w w_{\text{out}}}{2}
\]

(9)

Next disjunction and its reformulation through the convex hull technique \([33]\) is used for the optimal selection of fill type:

\[
y^1 + y^2 + y^3 = 1
\]

(10)

\begin{table}[h]
\centering
\caption{Constants for transfer coefficients.}
\begin{tabular}{ccc}
\hline
\( i \) & \( \alpha^i \) & \( j \) \\
\hline
1 & 0.249013 & 0.930306 & 0.101976 (splash fill) \\
2 & -0.464089 & -0.568230 & -0.432896 (trickle fill) \\
3 & 0.653578 & 0.641400 & 0.782744 (film fill) \\
4 & 0 & -0.352377 & -0.292870 \\
5 & 0 & -0.178670 & 0 \\
\hline
\end{tabular}
\end{table}

Here \( Y^1, Y^2 \) and \( Y^3 \) are Boolean variables used to select the splash fill, trickle fill or film fill, respectively. Therefore, when a Boolean variable is true its corresponding fill type is selected, but if it is false its corresponding fill type is not selected. To reformulate the above disjunction into algebraic expressions, the convex hull reformulation is used \([33]\), which includes a binary constraint to state that only one fill type (splash or trickle or film) can be selected.
where \( y^1, y^2 \) and \( y^3 \) are the corresponding binary variables of the Boolean variables. Then, if a Boolean variable is true its corresponding binary variable is equal to one; and if the Boolean variable is false, then its binary variable is zero. In addition, the value of the continuous variables \( (c_j) \) is expressed in terms of the disaggregated variables \( (c_j^1, c_j^2, c_j^3) \),

\[
c_j = c_j^1 + c_j^2 + c_j^3, \quad j = 1, \ldots, 5
\]

and the next equation allows to activate the values for the constants \( c_1, c_2, c_3, c_4 \) and \( c_5 \) depending on the fill type,

\[
c_j^1 = d_j^1 y^1, \quad i = 1, \ldots, 3, \quad j = 1, \ldots, 5
\]

Table 1 shows the values for the constants \( d_j \) for three types of packing [31].

The loss coefficient per meter depth of fill \( (K_f) \) is used to estimate the pressure drop through the packing region. The empirical correlation for \( K_f \) for different types of packing is expressed as [32],

\[
K_f = \left[ d_1 \left( \frac{m_{w,m}}{A_{m}} \right) d_2 \left( \frac{m_{a}}{A_{m}} \right) d_3 + d_4 \left( \frac{m_{w,m}}{A_{m}} \right) d_5 \left( \frac{m_{a}}{A_{m}} \right) d_6 \right] L_{fi}
\]

The correlation coefficients for the above relationship for each fill type are shown in Table 2. Following disjunction is used to select the value of the constants \( d_1, d_2, d_3, d_4, d_5 \) and \( d_6 \) depending on the fill type:

\[
\begin{cases}
  y^1 & \text{(splash fill)} \\
  d_k = d_k^1, \quad k = 1, \ldots, 6 & \\
  y^2 & \text{(trickle fill)} \\
  d_k = d_k^2, \quad k = 1, \ldots, 6 & \\
  y^3 & \text{(film fill)} \\
  d_k = d_k^3, \quad k = 1, \ldots, 6
\end{cases}
\]

Previous disjunction is reformulated with the convex hull reformulation [33] as follows:

\[
d_k = d_k^1 + d_k^2 + d_k^3, \quad k = 1, \ldots, 6
\]

where \( d_k^1, d_k^2 \) and \( d_k^3 \) are the disaggregated variables for the continuous variables \( d_k \). Values for the coefficients \( b_k \) for three different types of packing are shown in Table 2 [32].

3.2.3. Pressure drop of the air stream in the cooling tower

According to Li and Priddy [34], the total pressure drop \( (\Delta P_t) \) of the air stream in mechanical draft cooling towers is the sum of the static and dynamic pressure drops, \( \Delta P_{vp} \). The first one includes the pressure drop through the fill, \( \Delta P_f \), and the miscellaneous pressure losses, \( \Delta P_{misc} \). The pressure drop through the fill is calculated as [32],

\[
\Delta P_f = K_f A_{m} \left( \frac{\text{mav}^2}{2 \rho_{in} A_{m}^2} \right)
\]

In this equation, the harmonic mean density of the moist air through the fill, \( \rho_{in} \), and the arithmetic mean air–vapor mass flow rate, \( \text{mav}_{in} \), are calculated as follows:

\[
\text{mav}_{in} = \frac{\text{mav}_{in} + \text{mav}_{out}}{2}
\]

\[
\rho_{in} = \frac{1}{(1/\rho_{in} + 1/\rho_{out})}
\]

\[
\text{mav}_{in} = m_a + w_{in} m_a
\]

\[
\text{mav}_{out} = m_a + w_{out} m_a
\]

where \( \rho_{in}, \rho_{out}, \text{mav}_{in} \) and \( \text{mav}_{out} \) are the density of the inlet and outlet air, and the inlet and outlet arithmetic mean air–vapor mass flow rate, respectively.

The miscellaneous pressure drop can be directly calculated using the following equation [4,26]:

\[
\Delta P_{misc} = 6.5 \left( \frac{\text{mav}_{in}^2}{2 \rho_{in} A_{m}^2} \right)
\]

The dynamic pressure drop may be as much as 2/3 of the total static pressure drop [34]. Using such an upper value, the dynamic pressure drop can be estimated as

\[
\Delta P_{vp} = \frac{2}{3} \left( \Delta P_f + \Delta P_{misc} \right)
\]

Combining equations (16), (21) and (22), the total pressure drop of the air is given by

\[
\Delta P_t = 1.667 \left( \Delta P_f + \Delta P_{misc} \right)
\]

3.2.4. Power consumption

The power requirements for the cooling-tower fan (HP) can be calculated by multiplying the total pressure drop times the volumetric flow rate of the air stream, which depends on the localization of the fan. Therefore, for a forced mechanical draft wet-cooling tower, the power consumption can be calculated from [26]

\[
\text{HP} = \frac{\text{mav}_{in} \Delta P_t}{\rho_{in} \eta_f}
\]

where \( \eta_f \) is the fan efficiency.

3.2.5. Water consumption

In cooling towers, the water is lost due to evaporation (\( m_{w,ev} \)), drift (\( m_{w,d} \)), and blowdown (\( m_{w,b} \)). Conservation of mass yields the following relation for the evaporation rate of water

Table 2

<table>
<thead>
<tr>
<th>( k )</th>
<th>( b_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 3.179688 )</td>
</tr>
<tr>
<td>2</td>
<td>( 1.083916 )</td>
</tr>
<tr>
<td>3</td>
<td>( -1.965418 )</td>
</tr>
<tr>
<td>4</td>
<td>( 0.639008 )</td>
</tr>
<tr>
<td>5</td>
<td>( 0.649396 )</td>
</tr>
<tr>
<td>6</td>
<td>( 0.642767 )</td>
</tr>
<tr>
<td>( r^2 )</td>
<td>( 0.9932 )</td>
</tr>
</tbody>
</table>
\[ m_{w,\text{ev}} = m_d (w_{\text{out}} - w_{\text{in}}) \]  
(25)

The mass flow rate of blowdown can be expressed as [26]

\[ m_{w,b} = \frac{m_{w,r}}{n_{\text{cycle}}} - m_{w,d} \]  
(26)

where \( n_{\text{cycle}} \) is the number of cycles of concentration required to avoid salts deposition and typically takes a value between 2 and 4 [34]. In an efficient design, the water loss by drift is 0.2% of the inlet water flow rate to the cooling tower [35]

\[ m_{w,d} = 0.002 m_{w,\text{in}} \]  
(27)

Combining Equations (25)–(27), the fresh water consumption \( (m_{w,r}) \) is calculated as

\[ m_{w,r} = \frac{n_{\text{cycle}} m_{w,\text{ev}}}{n_{\text{cycle}} - 1} \]  
(28)

3.2.6. Feasibility constraints

Feasibility constraints and limits for the empirical correlations are included in the model to ensure feasible and valid designs.

In practice, the water outlet temperature should be at least 2.8 °C above site's wet-bulb temperature [34].

\[ T_{w,\text{out}} - T_{w,\text{in}} \geq 2.8 \]  
(29)

Since the governing equations are only valid for unsaturated air, a basic inequality constraint is that at no intermediate point in the packing the wet-bulb temperature of the air should be higher than the dry-bulb temperature of the air. This constraint is expressed as

\[ T_{a,n} \geq T_{w,\text{in}} + \Delta T_{T_{\text{wb,n}}}^{T_{\text{wb,n}}} \quad n \in N; \quad n \neq \text{NTI} \]  
(30)

where \( T_{a,n} \) is the dry-bulb temperature and \( T_{w,\text{in}} \) is the wet-bulb temperature of the air in the interval \( n \). \( \Delta T_{T_{\text{wb,n}}}^{T_{\text{wb,n}}} \) is a parameter that represents a minimum difference (i.e., 0.001) between above temperatures in each interval and it is used for avoiding the saturation of the air in an intermediate point of the packing height of the cooling tower.

The temperature of the water leaving the cooling tower must be lower than the coldest process temperature within the cooling water network [26].

\[ T_{w,\text{out}} \leq \text{TMPO} - \text{DTMIN} \]  
(31)

where TMPO is the outlet temperature of the coldest hot process stream in the cooling water network, and DTMIN is the minimum temperature difference allowed between hot process streams and cooling water. It should be noted that TMPO must be greater than \( T_{w,\text{in}} \).

The temperature of the water entering the cooling tower must be lower than the highest process temperature within the cooling water network [26].

\[ T_{w,\text{in}} \leq \text{TMPi} - \text{DTMIN} \]  
(32)

where TMPi is the inlet temperature of the hottest hot process stream in the cooling network.

At the same time, to avoid fouling, scaling and corrosion the inlet temperature of the water should not be greater than 50 °C [36].

\[ T_{w,\text{in}} \leq 50 \text{ °C} \]  
(33)

Practical bounds on the ratio of the mass flow rates \( m_{w}/m_a \) are given by [5]:

\[ 0.5 \leq \frac{m_{w,m}}{m_a} \leq 2.5 \]  
(34)

The empirical correlations for the transfer and loss coefficients are restricted by the following bounds on the water and air mass flow rates [31,32].

\[ 2.90 \leq \frac{m_{w,m}}{m_a} \leq 5.96 \]  
(35)

\[ 1.20 \leq \frac{m_a}{m_a} \leq 4.25 \]  
(36)

3.2.7. Objective function

The objective function of the MINLP model is to minimize the total annual cost, TAC, which is the sum of the annualized capital cost, CAP, and the operating cost, COP.

\[ \text{TAC} = K_f \text{CAP} + \text{COP} \]  
(37)

where \( K_f \) is the factor used to annualize the capital costs.

Fresh water (i.e. make-up water) consumption and power requirements determine the operating cost, which can be written as follows:

\[ \text{COP} = H_v c_{w,m} w_{r,t} + H_v c_{w,HP} \]  
(38)

where \( H_v \) is the yearly operating time, \( c_{w,m} \) is the unit cost of fresh water and \( c_{w,HP} \) is the unit cost of electricity.

The capital cost for the cooling tower is calculated through an empirical correlation that is expressed in terms of the fixed cooling-tower cost \( (C_{CTV}) \), the tower fill volume cost \( (C_{CTF}) \) and the tower cost based on air mass flow rate \( (C_{CTV}) \) [27].

\[ \text{CAP} = C_{CTF} + C_{CTV} A_{fl} + C_{CTV} m_a \]  
(39)

The cost term \( C_{CTV} \) depends on the type of packing; therefore, the next disjunction and its reformulation through the convex hull technique [33] is used:

\[ \begin{aligned}
C_{CTV} & = C_{CTV}^{1} + C_{CTV}^{2} + C_{CTV}^{3} \\
& = C_{CTV}^{1} + C_{CTV}^{2} + C_{CTV}^{3} \\
& = C_{CTV}^{1} + C_{CTV}^{2} + C_{CTV}^{3}
\end{aligned} \]  
(40)

\[ C_{CTV}^{i} = e^{i} \phi, \quad i = 1, ..., 3 \]  
(41)

Here \( C_{CTV}^{i}, C_{CTV}^{2} \) and \( C_{CTV}^{3} \) are the disaggregated variables of \( C_{CTV} \). Common values for the constants \( e^{i} \) are presented in Table 3.

The proposed MINLP model is given by equations (6)–(41) and the equations presented in the Appendix A and Appendix B. These relationships were codified in the GAMS optimization package [30] and the problem was solved using the solver DICOPT. It should be noted that the proposed MINLP problem is non-convex; therefore, the solutions that are obtained can be regarded only as local minima for the total annual cost problem depending of the starting point. Thus, it is important to account with good initial guesses for the optimization variables that can be obtained by simulation of

<table>
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<tr>
<th>( e^{i} )</th>
<th>( i = 1 )</th>
<th>( i = 2 )</th>
<th>( i = 3 )</th>
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<td>( C_{CTV} )</td>
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<td>( C_{CTV}^{3} )</td>
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4. Results and discussion

To show the application of the proposed model, six case studies taken from Serna-González et al. [26] were considered. For all examples, the values for the parameters \( H_y, K_y, H_{cycle}, \) \( c_{u, w}, c_{c, f}, C_{CTMA}, \) \( \eta, \Delta T^*/C^0 \), \( f_{gwo} \) and \( p_a \) are 8150 h/year, 0.2983 year\(^{-1} \), 4, 5.283 \times 10^{-4} \) USD/kg-water, 0.085 USD/kWh, 31,185 USD, 1097.5 USD \$/kg-dry-air, 0.75, 0.10, 2,501,598.5 J/K and 101,325 Pa, respectively.

In all calculations, if less than 20 intervals are used, the results become sensitive to the number of intervals. Therefore, the cooling-tower packing has been divided into 25 intervals to discretize the ordinary differential equations for all the case studies. With this number of intervals, the rigorous Poppe method gives the results shown in Table 4. Comparing these results with those obtained from the Merkel method [26], which also are shown in Table 4, we see that both methods select the film packing as the optimum type of filling material in all cases. Also, both methods provide for all cases optimal values of the outlet temperature of the water equal to the upper bound on \( T_{w,out} \) given by constraint (31). Therefore, there is no difference in the optimal values of the cooling-tower approach \((T_{w,out} - T_{wb,in})\) predicted by both models. It is worth noting that, in general, as the cooling-tower approach is reduced, the tower size and cost increase exponentially [26]. This explains the fact that, in all cases, the optimum values of the tower approach are given by the difference between the coldest process temperature of the cooling water network that the cooling tower is serving and the minimum temperature difference, i.e., \( T_{w,out} = \) TMPO – DTMIN.

All the cases show that using the Merkel method to solve the optimization problem results in an overestimation of the temperature of the air stream leaving the cooling tower. Taking case 1 as an example, it can be seen that the difference in \( T_{w,out} \) calculated by the Merkel method and the Poppe method is 24.38% of the Merkel value. The other cases present a similar behavior. The average difference in \( T_{w,out} \) for the six cases is 26.72% and the maximum difference is 41.66% (for case 5). This significant discrepancy is in large part due to the difference in the way that \( T_{w,out} \) is calculated in the two methods. As already mentioned above, the effect of water loss by evaporation is neglected when modeling the performance of cooling towers with Merkel method. As a result, in this approach only the outlet temperature of the water stream and the exit enthalpy of the air stream can be predicted. Nevertheless, the determination of the density and mass-fraction humidity of the outlet air is of prime importance when predicting the performance of the whole mechanical draft cooling unit [26]. The Merkel method, therefore, requires the additional assumption that the air at the top of the fill is saturated with water vapor to enable the approximate air outlet temperature to be calculated [6,26]. Then, this temperature is used to obtain the approximate exit density and mass-fraction humidity of the air. Knowing these properties, it is possible to estimate the total pressure drop of the air passing through the tower (Equation (23)) and the water loss by evaporation (Equation (25)) that are required for the optimal design of mechanical draft towers. It should be noted that the approximate outlet temperature of the air is calculated from the exit enthalpy that is obtained from an overall energy balance, which also ignores the effect of the reduction of water flow rate by evaporation [6,26]. In contrast to the procedure described above, when using the Poppe method for optimal design of a cooling tower, the full state of the outlet air in terms of enthalpy and mass-fraction humidity can be determined without using any of the simplifying assumptions of Merkel [6]. Thus, the optimal value of the air outlet temperature is very sensitive to the model used.

It is worthwhile to note here that the predicted and measured air outlet temperatures for several experimental fill performance tests are compared by Kloppers and Kröger [6]. These authors showed that the Merkel method might lead to significant discrepancies between predicted and experimental air outlet temperatures, especially when the actual outlet air is unsaturated, because of the several assumptions in which it relies. On the other hand, the air outlet temperatures predicted by the Poppe method agree very well with the field test results. This gives us an idea of how closely the Poppe model represents the conditions prevailing in a real wet-cooling tower. Furthermore, predictions from the Poppe formulation result in values of water evaporation rates that are in good agreement with full scale cooling-tower test results [24,25]. Thus, this model can be used with confidence during the optimal design of mechanical draft counter flow wet-cooling towers.

Our finding that the Poppe method gives lower optimal values of the wake-up water cost compared with the Merkel calculations for all the cases in Table 4 is thus not surprising. Two interrelated factors explain this decrease. First, a smaller temperature of the air leaving the fill reduces the mass-fraction humidity of that stream, which in turn reduces the value of \( w_{out} \) in \( (T_{w,out} - \) TMPO) in Equation (28), that is, the evaporation rates. For example, it can be seen for case 1 that \( w_{out} \) given by the Merkel method is 1.156 kg/s versus 0.8425 kg/s as determined by the Poppe method. Second, lower values of the fresh water consumption are associated with a decrease in the water loss by evaporation (Equation (28)).

Considering the inlet water mass flow rate for all the cases, the average optimal water loss by evaporation is 2.5% for the Poppe method and 4.34% for the Merkel method. The optimal make-up (i.e., fresh) water for the cooling tower is usually a little bit higher than this value for both methods because of the additional water loss due to blowdown and drift loss. Again, as noted previously, the difference in evaporated water mass flow rates estimated by the Poppe method and the Merkel method is due to the difference in the way that \( T_{w,out} \) is calculated, which in turn affects all optimal design and operation variables of wet-cooling towers such as the cooling range, total pressure drop of the air, tower height and so on.

As the calculations show for all the cases, both methods are consistent in predicting that the optimum performance occurs when the cooling-tower range \((T_{w,in} - T_{w,out})\) is greater than the tower approach. However, in contrast to the Merkel method that gives the higher hot water temperature, \( T_{w,in} = \) TMPI – DTMIN, it is interesting to see that the optimal values of the water inlet temperature predicted by the Poppe method are always lower than the upper bound on \( T_{w,in} \). This situation yields large differences in the optimal cooling ranges of the water calculated by the two methods. For example, the difference between the optimal cooling ranges predicted by the Merkel method and the Poppe method amounts to about 24% of the Merkel value for case 1. From the results in Table 4, one can obtain that the average of such difference is 41.14%, while the maximum difference is 73.86% for case 5. The large discrepancy between Merkel and Poppe cooling ranges in the cooling-tower optimization is mainly due to the large discrepancy in the air outlet temperature. For fixed heat load and water outlet temperature, this can be explained from the fact that a decrease in the air outlet temperature obtained by the Poppe method is associated with a lower exit enthalpy of the air and, thus, gives a lower water inlet temperature to satisfy the overall energy balance.

As can be seen from Table 4, some of the remaining results are fairly close in comparison and others are quite different. We have found that, in general, the Poppe method predicts greater values of the entering water mass flow rate and lower values of the water to dry-air mass flow ratio than the Merkel method. It should be
Table 4
Comparison of Poppe optimal solution with Merkel optimal solution for each case study.

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<td>29.0818</td>
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remembered that the value of $m_{w,m}/m_a$ reduces from the top to the bottom of the tower for Poppe method, whereas it is a constant for Merkel method. On the other hand, the increase in $m_{w,in}$ is simply due to the fact that the quantity of water circulated is inversely proportional to the cooling range for a given heat load (Equation (6)). Thus, in general, the reduction in the optimal cooling range given by the Poppe method yields an increase in the inlet water mass flow rate. In addition, since mechanical draft cooling towers are designed for $m_{w,m}/m_a$ ratios ranging from 0.5 to 2.5 (Equation (34)), the increase in $m_{w,in}$ also increases the optimal air mass flow rate given by the Poppe method. Thus, the Poppe method provides higher fan energy consumption and, consequently, fan power cost than the Merkel method. However, the operating costs (the make-up water cost plus the fan power cost) calculated by the Poppe method are lower than the ones obtained with the Merkel method except for case 5. Thus, in general, the lower values of evaporation rates and, consequently, the decrease in the make-up water cost predicted by the Poppe method has a more important impact on the operating costs than the increase in the air mass flow rate.

All the cases show that using the Merkel method results in the overestimation of the fill height. It can be seen in Table 4 that the average difference between the optimal fill height predicted by the Merkel method and the rigorous solutions of the Poppe model is 0.795 m for case 3 and the largest difference is 3.25% for case 5. The relative error, $(L_{AM} - L_{RG})/L_{AM}$, varies from 69.21% to 126.32%. Therefore, the errors in height of the tower packing caused by Merkel assumptions are significant. As a consequence, the Merkel method also predicts considerably higher Merkel numbers, or transfer characteristics, than the Poppe method in all cases, except for case 4 where the Merkel number is slightly over-predicted (3.15%) by the Merkel method compared with the Poppe method. The absolute average difference between the Merkel numbers, predicted by the two methods, is more than 38% with the Poppe method as a basis. Thus, in general, according to the Poppe model, the optimal cooling towers provided by the Merkel method are over-sized in terms of the Merkel number and fill height. One might argue, consequently, that the Merkel method could be of value in obtaining conservative optimal designs for mechanical draft wet-cooling towers.

It should be noted that the reduction in the Merkel number and packing height, according to the Poppe method, does not necessarily minimize the capital costs as it is shown for the cases 1, 2, 5 and 6 in Table 4. The reason for this result is the manner in which the capital costs are calculated. For preliminary designs, usually the capital cost model is a function only of the height of the tower packing. In situations like this, the cooling tower that features the minimum fill height will therefore have the minimum capital cost, even if it uses more fill area and air mass flow rate than others. However, in reality, the fill volume (which is the product of the height and the cross-sectional area of the tower packing) and the air mass flow rate play a role in the capital cost of a wet-cooling tower. For this reason we use a more detailed and more realistic correlation for capital costs that considers these factors (Equation (39)). Furthermore, as it was noted above, the decrease in the cooling range has the effect of increasing the entering water mass flow rate, with corresponding increases in the air mass flow rate and the fill volume for accomplishing the required cooling duty. This explains that the capital costs obtained with the Poppe method for the cases 1, 2, 5 and 6 are higher than those predicted by the Merkel method when the detailed costing method is used. It is remarkable that the capital costs of the cases 1 and 6, predicted by the two methods, are quite similar; however, the difference in the capital cost predicted by the Poppe method and the Merkel method is significant in the cases 2 (45.1%) and 5 (46.6%). The reason for this large difference in predicted capital costs is that the Merkel results for the cooling ranges of the cases 2 and 5 are considerably higher (213.9% and 282.6%, respectively) than those predicted by the Poppe method. For case 1, this large discrepancy in the cooling range can be explained by the reduction in the dry-bulb temperature of the inlet air at a constant wet-bulb temperature which, in turn, involves a decrease in potentials of the air for evaporation (i.e., $T_{a,in} - T_{wb,in}$ and $w_{s,w,in} - w_{in}$). On the other hand, for case 5 this is because the decrease in TMPO at a constant wet-bulb temperature of the entering air yields a reduction in the optimal tower approach, which also decreases the driving forces in the tower [26]. However, these situations are not properly handled by the Merkel method and, consequently, have a great influence on the relative difference between the optimal cooling ranges (i.e., outlet air temperatures) predicted by the Merkel and the Poppe methods.

As shown in Table 4, the Merkel predicted total annual cost is larger than the Poppe method predictions for the cases 1 (5.45%), 3 (16.39%), 4 (12.6%) and 6 (13.86%). On the other hand, for the cases 2 and 5, the predicted total annual cost is about 12% and 28% larger, respectively, in the Poppe method than the Merkel results, which again is due to the considerably larger optimal cooling range predicted by the Merkel method.

In the Merkel method the Lewis factor has been assumed to be unity. However, it can flow be observed in Table 4 that this is only approximately true. For the six cooling towers considered in this study, the resulting average Lewis factors from the Poppe method range between 0.9112 and 0.9268. For each case, the average Lewis factor is estimated as the average value between the inlet and outlet water conditions, $Lef_w = 0.5(Lef_{in} + Lef_{out})$. Therefore, according to this work, an assumed Lewis factor of about 0.92 may be more appropriate than unity. It is important to note that Sutherland [17] used a Lewis factor of 0.9 in his tower performance analysis; this figure is very close to the optimal average values of the Lewis factor predicted by the Poppe method for the examples in this work.

Each of the optimal designs calculated by the Merkel method was simulated using the rigorous Poppe method to assess its performance. Some results of the performance tests are shown in Fig. 4, in which for each case in Table 4 is plotted the number of integration intervals (i.e. height of the tower packing) versus the dry- and wet-bulb temperature of the air streams along the tower height. As can be seen from this figure for each case, the air temperatures increase continuously as they approach the top of the tower. It is interesting to see that at an intermediate point of the tower height there is an intersection of the $T_a$ and $T_{wb}$ curves, which indicates that the air stream reaches the point of saturation according to the Poppe method. In addition, above this point $T_{wb}$ is less than $T_a$ which indicates that the air stream is supersaturated according to the Poppe method. Therefore, the simulation results show that the air at the outlet of the fill is supersaturated for all the cases in Table 4. The assumption of Merkel that the outlet air is saturated with water vapor, regarding the calculation of the outlet air temperature, is thus not valid according to the Poppe method. This follows the conclusion presented above that the Merkel method does not predict accurately the air outlet temperature particularly when the ambient air gets warmer and drier [6]. Finally, it should be noted that usual preliminary design practice of cooling water systems is largely based on the minimum use of cooling water (i.e., the highest feasible cooling range), in an attempt to quickly obtain optimal configurations of these systems [37]. However, Ponce-Ortega et al. [2,38,39] have showed that only using the minimum water flow rate does not necessarily give the minimum total annual cost, since the capital costs of coolers and cooling towers must also be considered to find the correct trade-off between all the costs involved in the optimization of cooling water systems. This point is reinforced by the results obtained from the
Poppe method in this paper, in which the optimal cooling ranges are not consistent with heuristic for cooling water systems (i.e., optimal cooling-tower range should be as high as possible to use the minimum re-circulating water mass flow rate). Therefore, the proposed formulation could be placed in the context of integrated cooling water systems to develop a more reliable optimization framework for cooling systems.

5. Conclusions

In this paper, a mixer integer nonlinear programming model for the optimal detailed design of counter flow wet-cooling towers has been presented, in which the objective function consists in the minimization of the total annual cost considering the operation and capital costs of the towers. The rigorous Poppe method is used for estimating the size and performance of the tower, together with empirical correlations for the loss and overall mass transfer coefficients in the packing region of the cooling tower. The system of differential model equations that describes the cooling process is reduced to algebraic equations by using a fourth-order Runge–Kutta algorithm and dividing the cooling-tower packing into 25 integration intervals. It was found that further increase in this number of intervals did not cause any significant difference in the results.

The performance of the rigorous optimization formulation for six case studies was compared with that of a previously developed optimization algorithm based on the Merkel method [26]. We found that the Poppe method predicts the same optimal values of the tower approach than the Merkel method. However, for all examples, there are large differences between the remaining design and operation variables of wet-cooling towers (i.e., air outlet temperatures, evaporated water mass flow rates, cooling ranges, water and air mass flow rates, total pressure drops of air, evaporated water mass flow rates, Merkel numbers, and packing heights) predicted by the Poppe and Merkel methods. This is because the classical formulation of Merkel neglects the effect of the water loss by evaporation on the enthalpy change of the water stream and assumes a Lewis factor of unity and that the outlet air is saturated. Therefore, taking into consideration that the Poppe method allows a cooling tower to be described properly without resorting to the assumptions of the Merkel method, it is clear that it leads to more reliable optimal designs of wet-cooling towers.
Acknowledgements

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Appendix A. Thermophysical properties

We have used the following reliable air–water thermodynamic property equations in the rigorous model of the cooling tower that were taken from Kröger [40]. All temperatures are expressed in Kelvin.

- Enthalpy of the air–water vapor mixture per unit mass of dry-air

\[ \text{\(i_{ma} = cp_a(T_a - 273.15) + w[i_{fgwo} + cp_v(T_w - 273.15)] \)} \] (A.1)

- Enthalpy of the water vapor

\[ \text{\(i_v = i_{fgwo} + cp_{vw}(T_w - 273.15) \)} \] (A.2)

- Enthalpy of saturated air evaluated at water temperature

\[ \text{\(i_{ma,s,w} = cp_{a,w}(T_w - 273.15) + w_{s,w}[i_{fgwo} + cp_{vw}(T_w - 273.15)] \)} \] (A.3)

- Specific heat at constant pressure at \(T + 273.15/2\)

\[ \text{\(cp_a = 1.045356 \times 10^3 - 3.161783 \times 10^{-1}T + 7.083814 
\times 10^{-4}T^2 - 2.705209 \times 10^{-7}T^3 \)} \] (A.4)

- Specific heat of saturated water vapor at \(T + 273.15/2\)

\[ \text{\(cp_v = 1.3605 \times 10^3 + 2.31334T - 2.46784 \times 10^{-10}T^5 
+ 5.91332 \times 10^{-13}T^6 \)} \] (A.5)

- Latent heat for water at \(T = 273.15 \) K

\[ \text{\(i_{fgwo} = 3.4831814 \times 10^6 - 5.8627703 \times 10^3T + 12.139568T^2 
- 1.40290431 \times 10^{-2}T^3 \)} \] (A.6)

- Specific heat of water

\[ \text{\(cp_w = 8.15599 \times 10^3 - 2.80627 \times 10^7T + 5.11283 
\times 10^{-2}T^2 - 2.17582 \times 10^{-13}T^6 \)} \] (A.7)

- Humidity ratio

\[ \text{\(w = \left( \frac{2501.6 - 3.2363(T_{wb} - 273.15)}{2501.6 + 1.8577(T - 273.15) - 4.184(T_w - 273.15)} \right) \times \left( \frac{0.62509P_{v,wb}}{P_l - 1.005F_{v,wb}} \right) 
- \left( \frac{1.00416(T - T_{wb})}{2501.6 + 1.8577(T - 273.15) - 4.184(T_{wb} - 273.15)} \right) \)} \] (A.8)

- Vapor pressure

\[ \text{\(p_v = 10^2 \)} \] (A.9)

\[ \text{\(z = 10.79586 \left( 1 - \frac{273.16}{T} \right) + 5.02808 \log_{10} \left( \frac{273.16}{T} \right) + 1.50474 \)} \]
\[ \times 10^{-4} \left( 1 - 10^{-8.29692 \left( \frac{273.16}{T} - 1 \right)} \right) + 4.2873 \]
\[ \times 10^{-4} \left( 10^{4.76555 \left( \frac{273.16}{T} - 1 \right)} - 1 \right) + 2.786118312 \] (A.10)

- Density

\[ \text{\(\rho = \frac{P_l}{287.087} \left( 1 - \frac{w}{w + 0.62198} \right) \)} \] (A.11)

- Arithmetic mean air–vapor mass flow rate

\[ \text{\(m_{av} = m_a(1 + w) \)} \] (A.12)

Appendix B. System of nonlinear algebraic equations

Based on the work of Kloppers and Kröger [23], the differential equations of the Poppe model have been discretized using the fourth-order Runge–Kutta algorithm as described below:

It should be noted that the governing differential equations (Equations (1)–(3)) depend on the water temperature, the mass-fraction humidity and the air enthalpy. This can be represented as follows,

\[ \frac{dw}{dT_w} = f(i_{ma,w}, T_w) \] (B.1)

\[ \frac{di_{ma}}{dT_w} = f(i_{ma,w}, T_w) \] (B.2)

\[ \frac{dMe}{dT_w} = f(i_{ma,w}, T_w) \] (B.3)

According to the Runge–Kutta algorithm, the packing height of the cooling tower is divided into \(N\) finite intervals, with the same water temperature change (\(\Delta T_w\)) that is obtained by

\[ \Delta T_w = \frac{T_{w,in} - T_{w,out}}{N} \] (B.4)

The simultaneous discretization approach for optimization of wet-cooling towers is depicted in Fig. B1 for a cooling tower with five intervals. Furthermore, every interval is subdivided into three elements, as shown in Fig. B2 for interval 1. Using the nomenclature of these figures, the set of ordinary differential equations comprising the cooling-tower model is approximated at each interval by the following set of algebraic equations [23]:

\[ w_{n+1} = w_n + \left( f_n + 2f_{n+1} + 2f_{n+2} + f_{n+3} \right) / 6 \] (B.5)
Simultaneous discretization approach for optimizing counter flow wet-cooling towers using the Poppe method.

\[ i_{\text{ma},n+1} = i_{\text{ma},n} + \left( K_{n+1,1} + 2K_{n+1,2} + 2K_{n+1,3} + K_{n+1,4} \right) / 6 \]  
\[ M_{n+1} = M_{n} + \left( L_{n+1,1} + 2L_{n+1,2} + 2L_{n+1,3} + L_{n+1,4} \right) / 6 \]  

where

\[ J_{n+1,1} = \Delta T_{w} \cdot f(T_{w,\text{in}}, i_{\text{ma},n}, w_{n}) \]  
\[ K_{n+1,1} = \Delta T_{w} \cdot g(T_{w,\text{in}}, i_{\text{ma},n}, w_{n}) \]  
\[ L_{n+1,1} = \Delta T_{w} \cdot h(T_{w,\text{in}}, i_{\text{ma},n}, w_{n}) \]  
\[ J_{n+1,2} = \Delta T_{w} \cdot f\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,1}}{2}, w_{n} + \frac{J_{n+1,1}}{2} \right) \]  
\[ K_{n+1,2} = \Delta T_{w} \cdot g\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,1}}{2}, w_{n} + \frac{J_{n+1,1}}{2} \right) \]  
\[ L_{n+1,2} = \Delta T_{w} \cdot h\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,1}}{2}, w_{n} + \frac{J_{n+1,1}}{2} \right) \]  
\[ J_{n+1,3} = \Delta T_{w} \cdot f\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,2}}{2}, w_{n} + \frac{J_{n+1,2}}{2} \right) \]  
\[ K_{n+1,3} = \Delta T_{w} \cdot g\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,2}}{2}, w_{n} + \frac{J_{n+1,2}}{2} \right) \]  
\[ L_{n+1,3} = \Delta T_{w} \cdot h\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,2}}{2}, w_{n} + \frac{J_{n+1,2}}{2} \right) \]  
\[ J_{n+1,4} = \Delta T_{w} \cdot f\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,3}}{2}, w_{n} + \frac{J_{n+1,3}}{2} \right) \]  
\[ K_{n+1,4} = \Delta T_{w} \cdot g\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,3}}{2}, w_{n} + \frac{J_{n+1,3}}{2} \right) \]  
\[ L_{n+1,4} = \Delta T_{w} \cdot h\left( T_{w,\text{in}} + \frac{\Delta T_{w}}{2}, i_{\text{ma},n} + \frac{K_{n+1,3}}{2}, w_{n} + \frac{J_{n+1,3}}{2} \right) \]  

In above equations, \( J, K \) and \( L \) represent the recurrence relations to determine the changes of the air mass-fraction humidity, air enthalpy and Merkel number, respectively, corresponding to each incremental change in water temperature according to the fourth-order Runge–Kutta algorithm.

Equations (B.4)–(B.19) can be applied to each interval of the tower including the bottom (interval 1) and top (interval NTI) sections of the tower, where boundary conditions are given as:

\[ T_{w,\text{in}} = T_{w,n-\text{NTI}} \]  
\[ T_{w,\text{out}} = T_{w,n-0} \]  
\[ m_{w,\text{in}} = m_{w,n-\text{NTI}} \]  
\[ m_{w,\text{out}} = m_{w,n-0} \]  
\[ w_{\text{in}} = w_{n-0} \]
Variables

\[ W_{\text{out}} = W_{\text{n-NTI}} \]  \hspace{1cm} (B.25)
\[ T_{a,\text{in}} = T_{a,n-0} \]  \hspace{1cm} (B.26)
\[ T_{a,\text{out}} = T_{a,n-NTI} \]  \hspace{1cm} (B.27)
\[ T_{\text{wb,in}} = T_{\text{wb,n-0}} \]  \hspace{1cm} (B.28)
\[ T_{\text{wb,out}} = T_{\text{wb,n-NTI}} \]  \hspace{1cm} (B.29)
\[ i_{\text{ma,in}} = i_{\text{ma,n-0}} \]  \hspace{1cm} (B.30)
\[ i_{\text{ma,out}} = i_{\text{ma,n-NTI}} \]  \hspace{1cm} (B.31)

It should be noted that the properties of the water and air—
water vapor mixture and moist air, i.e., heat capacity, dry-bulb
temperature, wet-bulb temperature of air, mass-fraction humidity
or air, are needed at each interval of the cooling tower. These
properties are obtained from the property equations given in
Appendix A.

A set of simultaneous algebraic equations are thus created that
must be included in the proposed MINLP model for optimizing
mechanical draft wet-cooling towers.

Nomenclature

Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_f )</td>
<td>cross-sectional packing area, ( \text{m}^2 )</td>
</tr>
<tr>
<td>( \text{CAP} )</td>
<td>capital cost, ( \text{US$/year} )</td>
</tr>
<tr>
<td>( C_{\text{CTV}} )</td>
<td>disaggregated variables for the capital cost coefficients of cooling towers</td>
</tr>
<tr>
<td>( \text{COP} )</td>
<td>annual operating cost, ( \text{US$/year} )</td>
</tr>
<tr>
<td>( c_i )</td>
<td>variables for ( \text{Me} ) calculation</td>
</tr>
<tr>
<td>( c_i^j )</td>
<td>disaggregated variables for ( \text{Me} ) calculation</td>
</tr>
<tr>
<td>( c_{p_a} )</td>
<td>specific heat of air, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( c_{p_v} )</td>
<td>specific heat of saturated water vapor, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( c_{p_w} )</td>
<td>specific heat of liquid water, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( c_{p_{w,in}} )</td>
<td>specific heat of liquid water at the inlet of cooling tower, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( c_{p_{w,out}} )</td>
<td>specific heat of liquid water at the outlet of cooling tower, ( \text{J/kg K} )</td>
</tr>
<tr>
<td>( d_k )</td>
<td>variables used in the calculation of the loss coefficient</td>
</tr>
<tr>
<td>( d_k^j )</td>
<td>disaggregated variables for the calculation of the loss coefficient</td>
</tr>
<tr>
<td>( \text{HP} )</td>
<td>power fan, ( \text{hp} )</td>
</tr>
<tr>
<td>( i_{\text{ma}} )</td>
<td>enthalpy of the air—water vapor mixture per unit mass of dry-air, ( \text{J/kg dry-air} )</td>
</tr>
<tr>
<td>( i_{\text{ma,sw}} )</td>
<td>enthalpy of saturated air evaluated at water temperature, ( \text{J/kg dry-air} )</td>
</tr>
<tr>
<td>( i_v )</td>
<td>enthalpy of the water vapor, ( \text{J/kg dry-air} )</td>
</tr>
<tr>
<td>( J )</td>
<td>recurrence relation for calculating the incremental change of the air humidity</td>
</tr>
<tr>
<td>( K )</td>
<td>recurrence relation for calculating the incremental change of the air enthalpy</td>
</tr>
<tr>
<td>( K_f )</td>
<td>loss coefficient in the fill, ( \text{m}^{-1} )</td>
</tr>
<tr>
<td>( K_{\text{misc}} )</td>
<td>component loss coefficient, dimensionless</td>
</tr>
<tr>
<td>( L )</td>
<td>recurrence relation for calculating the incremental change of the Merkel number</td>
</tr>
<tr>
<td>( L_f )</td>
<td>fill height, ( \text{m} )</td>
</tr>
<tr>
<td>( \text{Lef} )</td>
<td>Lewis factor, dimensionless</td>
</tr>
<tr>
<td>( m_a )</td>
<td>air mass flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{\text{av}} )</td>
<td>inlet air—vapor flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{\text{av,n}} )</td>
<td>mean air—vapor flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{\text{av,out}} )</td>
<td>outlet air—vapor flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( \text{Me} )</td>
<td>Merkel number or transfer characteristic, dimensionless</td>
</tr>
<tr>
<td>( m_w )</td>
<td>water mass flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,b} )</td>
<td>mass flow rate of blowdown, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,d} )</td>
<td>mass flow rate of drift loss of water, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,ev} )</td>
<td>mass flow rate of evaporated water, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,in} )</td>
<td>inlet water mass flow rate to the cooling tower, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,m} )</td>
<td>average water mass flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,out} )</td>
<td>outlet water mass flow rate from the cooling tower, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( m_{w,r} )</td>
<td>make-up water mass flow rate, ( \text{kg/s} )</td>
</tr>
<tr>
<td>( P_v_{\text{wb}} )</td>
<td>saturated vapor pressure, ( \text{Pa} )</td>
</tr>
<tr>
<td>( T_a )</td>
<td>dry-bulb air temperature, °C or K</td>
</tr>
<tr>
<td>( \text{TAC} )</td>
<td>total annual cost, ( \text{US$/year} )</td>
</tr>
<tr>
<td>( T_a,n )</td>
<td>dry-bulb air temperature in the integration intervals, °C or K</td>
</tr>
<tr>
<td>( T_w )</td>
<td>water temperature, °C or K</td>
</tr>
<tr>
<td>( T_{\text{wb}} )</td>
<td>wet-bulb temperature of the air, °C or K</td>
</tr>
<tr>
<td>( T_{\text{wb,n}} )</td>
<td>wet-bulb temperature of the air in the integration intervals, °C or K</td>
</tr>
<tr>
<td>( T_{\text{wb,in}} )</td>
<td>water inlet temperature, °C or K</td>
</tr>
<tr>
<td>( T_{\text{wb,out}} )</td>
<td>water outlet temperature, °C or K</td>
</tr>
<tr>
<td>( w )</td>
<td>mass-fraction humidity of air stream, ( \text{kg of water/kg of dry-air} )</td>
</tr>
<tr>
<td>( w_{\text{in}} )</td>
<td>outlet mass-fraction humidity of air stream, ( \text{kg of water/kg of dry-air} )</td>
</tr>
<tr>
<td>( w_{s,w} )</td>
<td>saturated mass-fraction humidity at water temperature, ( \text{kg of water/kg of dry-air} )</td>
</tr>
<tr>
<td>( \Delta P_i )</td>
<td>total pressure drop of air, ( \text{Pa} )</td>
</tr>
<tr>
<td>( \Delta P_{vp} )</td>
<td>dynamic pressure drop, ( \text{Pa} )</td>
</tr>
<tr>
<td>( \Delta P_{fl} )</td>
<td>fill pressure drop, ( \text{Pa} )</td>
</tr>
<tr>
<td>( \Delta P_{\text{misc}} )</td>
<td>miscellaneous pressure drop, ( \text{Pa} )</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>harmonic mean density of air—water vapor mixtures, ( \text{kg/m}^3 )</td>
</tr>
<tr>
<td>( \rho_{\text{out}} )</td>
<td>outlet air density, ( \text{kg/m}^3 )</td>
</tr>
</tbody>
</table>

Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i^j )</td>
<td>disaggregated coefficients for the estimation of ( \text{Me} )</td>
</tr>
<tr>
<td>( b_k )</td>
<td>disaggregated coefficients for the estimation of loss coefficient</td>
</tr>
<tr>
<td>( c_1-c_5 )</td>
<td>correlation coefficients for the estimation of ( \text{Me} )</td>
</tr>
<tr>
<td>( C_{\text{CTF}} )</td>
<td>fixed cooling-tower cost, ( \text{US$} )</td>
</tr>
<tr>
<td>( C_{\text{CTVM}} )</td>
<td>incremental cooling-tower cost based on air mass flow rate, ( \text{US$/kg} )</td>
</tr>
<tr>
<td>( C_{\text{CTV}} )</td>
<td>incremental cooling-tower cost based on tower fill volume, ( \text{US$/m}^3 )</td>
</tr>
<tr>
<td>( c_{\text{ue}} )</td>
<td>unit cost of electricity, ( \text{US$/kW-h} )</td>
</tr>
<tr>
<td>( c_{\text{uw}} )</td>
<td>unit cost of make-up water, ( \text{US$/kg} )</td>
</tr>
<tr>
<td>( d_{1-6} )</td>
<td>correlation coefficients for the estimation of loss coefficient, dimensionless</td>
</tr>
<tr>
<td>( D_{\text{TMIN}} )</td>
<td>minimum allowable temperature difference, °C or K</td>
</tr>
<tr>
<td>( e_i )</td>
<td>coefficient cost for different fill types</td>
</tr>
<tr>
<td>( H_y )</td>
<td>yearly operating time, ( \text{hr/year} )</td>
</tr>
<tr>
<td>( h_{\text{gwo}} )</td>
<td>latent heat of water, ( \text{J/kg} )</td>
</tr>
<tr>
<td>( K_{\text{F}} )</td>
<td>annualization factor, ( \text{year}^{-1} )</td>
</tr>
<tr>
<td>( n_{\text{cycles}} )</td>
<td>number of cycles of concentration, dimensionless</td>
</tr>
<tr>
<td>( P_t )</td>
<td>total pressure, ( \text{Pa} )</td>
</tr>
<tr>
<td>( Q )</td>
<td>heat load, ( \text{W or kW} )</td>
</tr>
<tr>
<td>( T_{a,in} )</td>
<td>dry-bulb air temperature of the air entering the cooling tower, °C or K</td>
</tr>
<tr>
<td>( \text{TMPI} )</td>
<td>inlet temperature of the hottest hot process stream, °C or K</td>
</tr>
<tr>
<td>( \text{TMPO} )</td>
<td>outlet temperature of the hottest hot process stream, °C or K</td>
</tr>
</tbody>
</table>
The wet-bulb temperature of the air entering the cooling tower, °C or K

inlet mass-fraction humidity of air stream, kg of water/kg of dry-air

fan efficiency, dimensionless

inlet air density, kg/m³

Binary variables

\( \gamma^k \)

used to select the type of fill

Subscripts

a dry-air

b blowdown
d drift
e electricity
ev evaporated water
ff fan
g packing or fill
fr cross-sectional
i inlet
j constants for calculating the transfer coefficient depending on the fill type, \( j = 1, \ldots, 5 \)
k constants for the loss coefficient depending on the fill type, \( k = 1, \ldots, 6 \)
m average

ma air–vapor mixture

misc miscellaneous

n number of interval

out outlet

r make-up (i.e. fresh)
s saturated
t total
v water vapor
vp velocity pressure
w water

wb wet-bulb temperature

Superscripts

i fill type, \( i = 1, 2, 3 \)
j constants in the fill type, \( j = 1, \ldots, 5 \)
k constants in the fill type, \( k = 1, \ldots, 6 \)

References


